

Physics 352 – Part 2 – Statistical Mechanics

Study Guide for the Final Exam

NOTE: This list is intended only as an approximate guide to the topics with which you should be familiar, in preparation for the upcoming exam. There will certainly be topics listed here that are not present on the exam. And of course, this list won't spell out exactly everything that will show up in the midterm – it is meant only as an approximate guide.

Format of the exam: The exam (Mon. March 17) will be closed book / closed notes, and 2 hours long. Some information will be supplied to you (see below).

General advice: Study your homework assignments and notes. Be sure to study the *solutions* to homework problems you didn't understand. Understand the *derivations* of everything, not just the results. In statistical mechanics, it may seem like there are many things to remember – this is emphatically **not true**. There are **very few** important things to remember – but many things that you should be able to derive given these starting points. See below.

IMPORTANT THINGS YOU SHOULD KNOW AND REMEMBER:

- The fundamental axiom of statistical mechanics, and what it means.
- Thermal equilibrium: the relation $\tau_1 = \tau_2$, and how we derived it.
- The Boltzmann Distribution $p_i = \frac{1}{Z} \exp\left(-E_i/\tau\right)$, and how to use it.
- The partition function $Z = \sum_i \exp\left(-E_i/\tau\right)$, and how to use it.
- How to calculate the average value of any property.
- The relation $P_1 = P_2$, and how we derived it.
- The thermodynamic identity $\tau d\sigma = dU + PdV$, or $dQ = dU + dW$, and what it means.
- The Helmholtz Free Energy $F = U - \tau\sigma$, and what it means.

YOU NEED NOT REMEMBER, BUT YOU SHOULD KNOW HOW TO DERIVE:

- Expressions like $U = -\frac{1}{Z} \left(\frac{\partial}{\partial \beta} Z \right)$ and $U = \tau^2 \frac{1}{Z} \left(\frac{\partial}{\partial \tau} Z \right)$ (for the mean energy).
- Expressions like $P = -\sum_i p_i \left. \frac{\partial E_i}{\partial V} \right|_{\sigma}$, given “starting points” like $P = -\left. \frac{\partial U}{\partial V} \right|_{\sigma}$; also, know how this leads to $P = \frac{2}{3} \frac{U}{V}$ for any system for which $E \propto V^{-2/3}$.

- Expressions like $P = -\left.\frac{\partial F}{\partial V}\right|_{\tau}$ and $\sigma = -\left.\frac{\partial F}{\partial \tau}\right|_V$, which follow from considering “ dF ”.
- The observation that $dF = -dW$ at constant temperature.
- Expressions for the heat capacity of an ideal gas at constant volume or pressure.
- Expressions describing the consequences of expansion (i.e. changes in volume) for an ideal gas at constant temperature or entropy.
- The ideal (Carnot) efficiency of a heat engine.

YOU WILL BE GIVEN:

- The energy level structure of any particular systems examined.
- Various useful relations, listed below.

NOTATION:

P = pressure; τ = temperature (absolute, “fundamental” temperature); T = conventional temperature; β = inverse temperature = $1/\tau$; E = energy; U = mean energy; σ = entropy; V = volume; F = Helmholtz Free Energy; Z = partition function; N = number of particles

RELATIONS:

- Binomial Distribution Formula: the number of combinations of k “up” spins out of N spins is
$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$
- Stirling’s approximation for factorials: $\ln(N!) \approx N \ln(N) - N$.
- Geometric series:
$$\sum_{m=0}^{\infty} r^m = \frac{1}{1-r}$$
- Pressure is defined by $\frac{P}{\tau} = \left.\frac{\partial \sigma}{\partial V}\right|_{E,N}$. Another relation:
$$P = -\left.\frac{\partial U}{\partial V}\right|_{\sigma}$$
- $F = -\tau \ln(Z)$
- Energy of a Monatomic Ideal Gas:
$$U = \frac{3}{2} N \tau$$
- Entropy of a Monatomic Ideal Gas:
$$\sigma = N \left(\frac{5}{2} + \ln \left(\frac{n_Q V}{N} \right) \right)$$
- The quantum concentration for a particle of mass m :
$$n_Q = \left(\frac{m \tau}{2\pi \hbar^2} \right)^{3/2}$$
- Specific heat capacity at constant volume:
$$c_V \equiv \left. \frac{dQ}{d\tau} \right|_V$$
- Specific heat capacity at constant pressure:
$$c_P \equiv \left. \frac{dQ}{d\tau} \right|_P$$